Lesson 27: Recursive Challenge Problem—The Double and Add 5 Game

The *double and add 5* game is loosely related to the Collatz conjecture—an unsolved conjecture in mathematics named after Lothar Collatz, who first proposed the problem in 1937. The conjecture includes a recurrence relation, triple and add 1, as part of the problem statement. It is a worthwhile activity for you to read about the conjecture online.

Classwork

**Example**

Recall Exercise 3 from the previous lesson: Using a generic initial value, , and the recurrence relation, , for , find a formula for , , , in terms of .

Mathematical Modeling Exercise/Exercise 1

Using one of the four formulas from Example 1, write an inequality that, if solved for , will lead to finding the smallest starting whole number for the *double and add* *5* game that produces a result of or greater in rounds or fewer.

Exercise 2

Solve the inequality derived in Exercise 1. Interpret your answer, and validate that it is the solution to the problem. That is, show that the whole number you found results in or greater in three rounds, but the previous whole number takes four rounds to reach .

Exercise 3

Find the smallest starting whole number for the *double and add 5* game that produces a result of or greater in four rounds or fewer.

Lesson Summary

The formula, , describes the th term of the *double and add 5* game in terms of the starting number and . Use this formula to find the smallest starting whole number for the *double and add 5* game that produces a result of or greater in rounds or fewer.

Problem Set

1. Your older sibling came home from college for the weekend and showed you the following sequences (from her homework) that she claimed were generated from initial values and recurrence relations. For each sequence, find an initial value and recurrence relation that describes the sequence. (Your sister showed you an answer to the first problem.)
2. Answer the following questions about the recursive sequence generated by initial value, , and recurrence relation, for .
   1. Find a formula for , , , , in terms of powers of .
   2. Your friend, Carl, says that he can describe the th term of the sequence using the formula, . Is Carl correct? Write one or two sentences using the recurrence relation to explain why or why not.
3. The formula, , describes the th term of the *double and add 5* game in terms of the starting number and . Verify that it does describe the th term by filling out the tables below for parts (b) through (e). (The first table is done for you.)
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1. Bilbo Baggins stated to Samwise Gamgee, “Today, Sam, I will give you . Every day thereafter for the next days, I will take the previous day’s amount, double it and add , and give that new amount to you for that day.”
   1. How much did Bilbo give Sam on day ? (Hint: You don’t have to compute each term.)
   2. Did Bilbo give Sam more than altogether?
2. The formula, , describes the th term of the *double and add 5* game in terms of the starting number and . Use this formula to find the smallest starting whole number for the *double and add 5* game that produces a result of or greater in rounds or fewer.